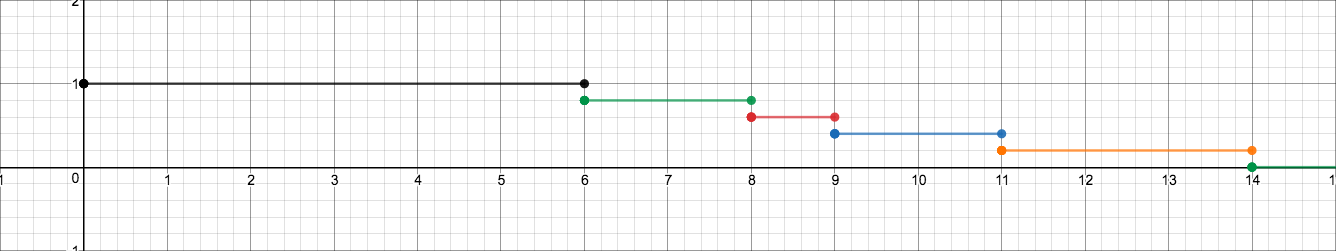
1. Let be independent and identically distributed random variables, whose common mean is and common variance . Let denote the sample mean, that is, .  
   1. Prove that .  
        
         
         
        
      However,  
        
      Therefore,
   2. Prove that .   
        
      However,  
        
      Therefore,
2. Let be independent and identically distributed random variables, whose common mean is and common variance . With denoting the sample mean, let be the residual sum of squares, that is, .  
   1. Prove that .
   2. Prove that , where is a sample variance, which is defined by the equation , that is, .
3. Let be independent and identically distributed times, whose common reliability function is . Let denote the empirical reliability function, which means that is the proportion of those that exceed .  
   1. Suppose that the observed failure times of five identically manufactured wind turbines are 14, 6, 8, 11, and 9 months. Draw the empirical reliability function .  
        
      
   2. Let denote the random variable that takes the value 1 when the event occurs and 0 otherwise. Prove that is a Bernoulli random variable, and express the probability of the event in terms of the aforementioned common reliability function.
   3. Prove that can be written as the sample mean of the Bernoulli random variables , where .
   4. Calculate the mean .
   5. Calculate the variance .  
        
         
        
      Therefore,  
        
      And,
4. Let be independent and identically distributed random variables, whose common distribution function is . Let denote the empirical distribution function, which means that is the proportion of those that do not exceed .  
   1. Let denote the random variable that takes the value 1 when the event occurs and 0 otherwise. Prove that is the Bernoulli random variable and express the probability of the event in terms of the aforementioned common distribution function.
   2. Prove that can be written as the sample mean of the Bernoulli random variables , where .
   3. Calculate the mean .
   4. Calculate the variance .  
        
         
        
      Therefore,  
        
      And,
5. 1. Prove Markov’s Inequality, which says that, for every constant and every non-negative random variable , the probability does not exceed .  
        
      Let be a RV such that,   
         
        
      Since , then
   2. Use Markov’s Inequality to prove Tchebychev’s Inequality, which says that, for every constant and every random variable with mean and finite variance , the probability does not exceed .  
        
       is equal to , therefore,
   3. Weak Law of Large Numbers (WLLN): Let be independent and identically distributed random variables with common mean and (finite) common variance . Prove that the sample mean approaches the (theoretical) mean in probability, meaning that no matter what constant you choose, you have when .  
        
      Because we are dealing with the empirical variance,   
        
      Tchebychev’s Inequality,  
        
      , , and
   4. Let be independent Bernoulli random variables, each having the same probability of success , that is, for every . Prove that the proportion of successes among these Bernoulli random variables approaches the (theoretical) proportion in probability when gets larger and larger.  
        
      Using the Tchebychev’s Inequality,  
        
      , , and
   5. Let be independent and identically distributed failure times, whose common reliability function is . Let denote the empirical reliability function. Prove that approaches in probability when gets larger and larger.  
        
      Using the Tchebychev’s Inequality,  
      , , and
   6. Let be independent and identically distributed failure times, whose common distribution function is . Let denote the empirical reliability function. Prove that approaches in probability when gets larger and larger.  
        
      Using the Tchebychev’s Inequality,  
      , , and